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EFFECT OF TEMPERATURE CHANGES ON DAMPING PROPERTIES OF SANDWICH CYLINDRICAL PANELS

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Abstract— The effect of changing temperature on the damping properties of free vibrations of sandwich cylindrical panels is studied in this paper. It is assumed that the material properties of the facings and core of the sandwich panel studied change with temperature. The effects of the viscoelastic damping of the core layer and geometrical non-linearities of the vibrating panel have been included in this study. Dissipated energy is used to investigate the damping. It is found that the rapidly changing temperature strongly affects the vibration responses and damping properties of the cylindrical sandwich panels.

NOTATION

a, b	length and arc length in the x and y directions, respectively
E_i	elastic modulus of the ith layer
G_i	transverse shear modulus of the core in the i direction
$i = 1, 2, 3$	subscript corresponding to lower facing, core and upper facing, respectively
R	radius of reference plane
t_i	thickness of the ith layer
u_i, v_i, w_i	displacement in the ith layer in the x, y and z directions, respectively
μ_i	Voigt damping parameters in the viscoelastic core
ν	Poisson's ratio
ρ_i	mass density of the ith layer
ϕ, ψ	slope of the core in the x and y directions, respectively

1. INTRODUCTION

Sandwich panels are widely used in the aerospace industry and in nuclear reactors. These panels are often applied to insulate the structure from external temperature changes. The change of temperature usually results in the change of the damping and elastic properties of the sandwich panels and therefore in their vibration responses. A review related to thermal effects on structures can be found in a recent paper by Thornton (1992) which describes development of thermal structures of aeronautical and aerospace industries from the early days of supersonic airplanes to the more recent challenges presented by hypersonic fighters.

It is well known that the amplitudes of structural vibrations can be reduced by using layers of viscoelastically damped materials. The effect of the viscoelasticity on vibrating sandwich plates was first studied by Oberst and Frankenfeld (1952), who considered the one-dimensional case of an infinite, two layer plate, in which one layer is made of damping material. The earliest work on the damping of ordinary sandwich plate, by Plass (1957), proved that the damping is entirely due to the shear in the viscoelastic core. Since then many papers have been published on the damped vibrations of sandwich plates (Yu, 1962; Ho and Łukasiewicz, 1975; He and Ma, 1988). Ho and Łukasiewicz (1976) also investigated the cylindrical sandwich shells with dissimilar facings. The concept of the complex modulus was used to analyse the damping properties of the structure.

Only a few studies have been devoted to the non-linear analysis of free damped vibrations of sandwich structures. Kovac *et al.* (1971) and Hyer *et al.* (1976, 1978) studied non-linear vibrations of damped sandwich beams using both theoretical and experimental methods. Xia and Łukasiewicz (1994, 1995) analysed damping properties of largely deformed sandwich plates using the logarithmic decrement and dissipated energy for the transverse motion.

Temperature is usually considered to be the most important factor affecting the properties of damping materials (Jones, 1974). Nashif *et al.* (1985) studied the relationship between the temperature and the properties of different damping materials. Gorman (1985) carried out a linear vibration study on a composite plate which had temperature-dependent properties. Less attention has been paid to the analysis of the effects of temperature on the dynamic response of structures, including damping. The recent work by Łukasiewicz and Xia (1993, 1995) and Yi *et al.* (1993) discussed the effects of temperature on the dynamic response of sandwich plates. The purpose of the present paper is to study the effects of rapidly changing temperature on damping properties of non-linear vibrations of sandwich cylindrical panels. The dissipated energy of the structure is analysed and used as a measurement of damping. The applied model of deformations allows bending, transverse shear deformations and rotatory inertia to be included in the analysis. Damping is considered by modelling the viscoelastic material core as a Voigt–Kelvin solid. The Runge–Kutta method is employed to solve the non-linear, differential equations.

2. NON-LINEAR GOVERNING EQUATIONS

The dimensions and orthogonal coordinate system of the panel chosen for this study are shown in Fig. 1. The coordinates x and y are in the reference surface of the undeformed panel and the z coordinate is oriented in the direction perpendicular to this surface. The reference surface is assumed to be in the middle of the core. The corresponding displacement components are u_i, v_i and w_i, with i used to define the displacement of the ith layer.

The assumptions made in the present study are as follows. Each layer is bonded together perfectly and there is no slide between the interfaces. Layers 1 and 3 are isotropic and made of perfectly elastic material. Layer 2 consists of a viscoelastic material which is able to carry the transverse shear stress. Lines which are normal to the reference surface of the undeformed core remain straight after deformation, but they do not remain normal to the reference surface of the panel. The Kirchhoff hypothesis is used only for the facings.

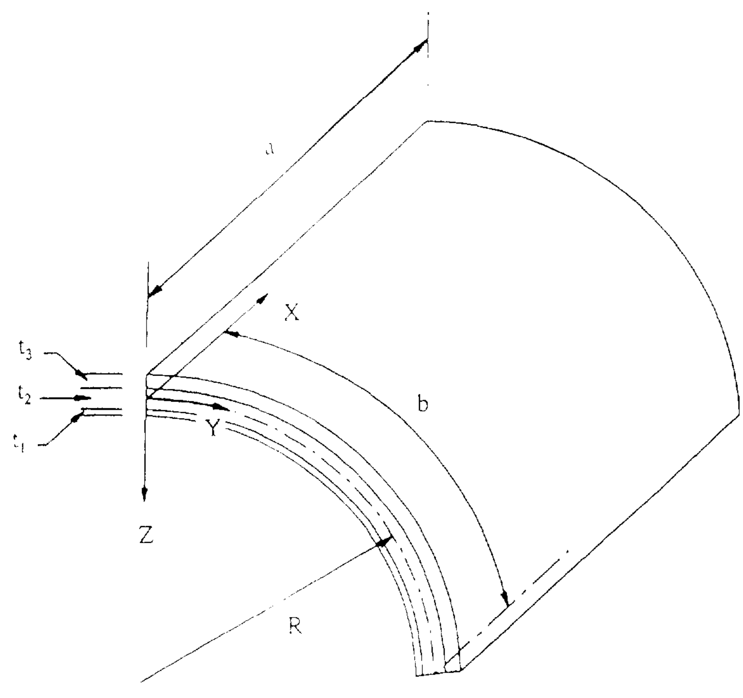

Fig. 1. Geometrical dimensions and coordinate system of sandwich cylindrical panels.

Compression effects are neglected. Normal strains in the thickness direction are ignored. Transverse displacement is assumed to be the same for each layer.

Using the above assumptions, the displacements in each layer (see Fig. 1) can be expressed in terms of five variables u_2^0, v_2^0, ϕ, ψ and w as follows:

$$
\begin{aligned}
u_1 &= u_2^0 - t_2(\phi - w_{,x})/2 - z w_{,x} \\
v_1 &= v_2^0 - t_2(\psi - w_{,y})/2 - z w_{,y} \\
u_3 &= u_2^0 + t_2(\phi - w_{,x})/2 - z w_{,x} \\
v_3 &= v_2^0 + t_2(\psi - w_{,y})/2 - z w_{,y} \\
u_2 &= u_2^0 - z\phi \\
v_2 &= v_2^0 - z\psi \\
w_i &= w,
\end{aligned}
\tag{1}
$$

where u_2^0, v_2^0 are the in-plane displacements in the reference surface of the panel, ϕ and ψ are the rotatory angles of the core in the x and y directions, respectively, and w is the transverse deflection in the z direction in three layers. The non-linear strain–displacement relations of elastic facings are

$$
\begin{aligned}
\varepsilon_{1x} &= u_{,x} - \tfrac{1}{2}t_2\phi_{,x} + (\tfrac{1}{2}t_2 - z)w_{,xx} + \tfrac{1}{2}w_{,x}^2 \\
\varepsilon_{1y} &= v_{,y} - \tfrac{1}{2}t_2\psi_{,y} + (\tfrac{1}{2}t_2 - z)w_{,yy} - \frac{1}{R-z}w + \tfrac{1}{2}w_{,y}^2 \\
\varepsilon_{1xy} &= u_{,y} + v_{,x} - \tfrac{1}{2}t_2(\phi_{,y} + \psi_{,x}) + 2(\tfrac{1}{2}t_2 - z)w_{,xy} + w_{,x}w_{,y} \\
\varepsilon_{3x} &= u_{,x} + \tfrac{1}{2}t_2\phi_{,x} - (\tfrac{1}{2}t_2 + z)w_{,xx} + \tfrac{1}{2}w_{,x}^2 \\
\varepsilon_{3y} &= v_{,y} + \tfrac{1}{2}t_2\psi_{,y} - (\tfrac{1}{2}t_2 + z)w_{,yy} - \frac{1}{R-z}w + \tfrac{1}{2}w_{,y}^2 \\
\varepsilon_{3xy} &= u_{,y} + v_{,x} + \tfrac{1}{2}t_2(\phi_{,y} + \psi_{,x}) - 2(\tfrac{1}{2}t_2 + z)w_{,xy} + w_{,x}w_{,y} \\
\varepsilon_{2x} &= u_{,x} - z\phi_{,x} + \tfrac{1}{2}\phi^2 \\
\varepsilon_{2y} &= v_{,y} - z\psi_{,y} - \frac{1}{R-z}w + \tfrac{1}{2}\psi^2 \\
\varepsilon_{2xy} &= u_{,y} + v_{,x} - z(\phi_{,y} + \psi_{,x}) + \phi\psi \\
\varepsilon_{2xz} &= w_{,x} - \phi \\
\varepsilon_{2yz} &= w_{,y} - \psi.
\end{aligned}
\tag{2}
$$

In eqns (2), a comma denotes partial differentiation with respect to the corresponding coordinates. For simplicity, the superscript 0 and subscript 2 of u_2^0 and v_2^0 are ignored in eqns (2) and the remainder of the paper. The transverse shear strains in the facings have been neglected.

The change of temperature causes a thermal expansion of each layer of the panel. After neglecting the influence of transverse normal strain, the relationship between the stresses and deformations for the facings have the following form when thermal stresses are included (Łukasiewicz, 1989):

$$
\begin{aligned}
\sigma_{ix} &= \frac{E_i}{1-\nu_i^2}(\varepsilon_{ix} + \nu_i\varepsilon_{iy}) - \frac{E_i\alpha_{ti}}{1-\nu_i}T_i \\
\sigma_{iy} &= \frac{E_i}{1-\nu_i^2}(\varepsilon_{iy} + \nu_i\varepsilon_{ix}) - \frac{E_i\alpha_{ti}}{1-\nu_i}T_i \\
\sigma_{ixy} &= \frac{E_i}{2(1+\nu_i)}\varepsilon_{ixy}.
\end{aligned}
\tag{3}
$$

where α_{ti} are the thermal coefficients and T_i are temperature changes in the ith layer.

For the viscoelastic core, the total stress consists of two parts

$$\sigma^{t}_{2ij} = \sigma_{2ij} + \sigma^{d}_{2ij}, \tag{4}$$

where σ_{ij} is an elastic stress and σ^{d}_{ij} is defined as the stress from viscoelastic damping. When the Voigt–Kelvin model is used, the stress–strain relationship takes the form:

$$\begin{aligned}
\sigma_{2xx} &= \frac{E_2}{1-v_2^2}(\varepsilon_{2xx}+v_2\varepsilon_{2yy}) - \frac{E_2\alpha_{t2}}{1-v_2}T_2, \quad \sigma^{d}_{2xx} = \frac{E_2}{1-v_2^2}\mu_x(\dot\varepsilon_{2xx}+v_2\dot\varepsilon_{2yy}),\\
\sigma_{2yy} &= \frac{E_2}{1-v_2^2}(\varepsilon_{2yy}+v_2\varepsilon_{2xx}) - \frac{E_2\alpha_{t2}}{1-v_2}T_2, \quad \sigma^{d}_{2yy} = \frac{E_2}{1-v_2^2}\mu_y(\dot\varepsilon_{2yy}+v_2\dot\varepsilon_{2xx}),\\
\sigma_{2xy} &= \frac{E_2}{2(1+v_2)}\varepsilon_{2xy}, \quad \sigma^{d}_{2xy} = \frac{E_2}{2(1+v_2)}\mu_{xy}\dot\varepsilon_{2xy},\\
\sigma_{2xz} &= G_{2xz}\varepsilon_{2xz}, \quad \sigma^{d}_{2xz} = G_{2xz}\mu_{xz}\dot\varepsilon_{2xz},\\
\sigma_{2yz} &= G_{2yz}\varepsilon_{2yz}, \quad \sigma^{d}_{2yz} = G_{2yz}\mu_{yz}\dot\varepsilon_{2yz}.
\end{aligned} \tag{5}$$

μ_{ij} are the damping parameters determined from experiments, which can also be found, for example, in the book by Nashif *et al.* (1985). $\dot\varepsilon$ are the strain rates in the second layer core. Relationships (3) and (5) consider thermal stress by including additional terms that represent the effect of the temperature change T_i in the ith layer. The thermal coefficients α_{ti} and material properties E_i and μ_i are assumed to be functions of temperature and time.

The dynamic virtual work principle for the thermoelastic body requires that

$$\delta\Pi^{t} = \delta\Pi + \delta\Pi^{d} = 0, \tag{6}$$

where

$$\delta\Pi = \int_t\int_x\int_y\left(\int_z\left(\sum_{i=1}^{3}((\sigma_{ix}\delta\varepsilon_{ix}+\sigma_{iy}\delta\varepsilon_{iy}+\sigma_{ixy}\delta\varepsilon_{ixy}) - \rho_i(\ddot u_i\delta u_i+\ddot v_i\delta v_i+\ddot w\delta w)) + (\sigma_{2xz}\delta\varepsilon_{2xz}+\sigma_{2yz}\delta\varepsilon_{2yz})\right)\left(1-\frac{z}{R}\right)\mathrm{d}z + p(x,y,t)\delta w\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}t \tag{7}$$

presents the elastic energy and the kinetic energy of the panel. The dissipated energy takes the form

$$\delta\Pi^{d} = \int_t\int_x\int_y\int_z(\sigma^{d}_{2x}\delta\varepsilon_{2x}+\sigma^{d}_{2y}\delta\varepsilon_{2y}+\sigma^{d}_{2xy}\delta\varepsilon_{2xy}+\sigma^{d}_{2xz}\delta\varepsilon_{2xz}+\sigma^{d}_{2yz}\delta\varepsilon_{2yz})\left(1-\frac{z}{R}\right)\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t; \tag{8}$$

in eqn (7) ρ_i is the mass density per unit volume of the ith layer and $p(x, y, t)$ is the transverse load per unit area. If we limit ourselves to the study of free vibrations, $p(x, y, t)$ is assumed to be equal to zero. With the usual integration and variational procedures, eqn (6) can be reduced to a system of differential equations in terms of five variables u_2, v_2, ϕ, ψ and w as presented in the following:

$$K_1(2U_{,\xi\xi} + (1-\nu)\alpha_1^2 U_{,\eta\eta}) + K_2 V_{,\xi\eta} + K_3(2\Phi_{,\xi\xi} + (1-\nu)\alpha_1^2\Phi_{,\eta\eta}) + \alpha_1 K_4 \Psi_{,\xi\eta} + K_5(W_{,\xi\xi\xi}$$
$$+ \alpha_1^2 W_{,\xi\eta\eta}) + \nu K_6 W_{,\xi} - K_7(2\mu_x \dot U_{,\xi\xi} + (1-\nu)\alpha_1^2\mu_{xy}\dot U_{,\eta\eta}) + \alpha_1 K_7(2\nu\mu_y + (1-\nu)\mu_{xy})\dot V_{,\xi\eta}$$
$$+ K_8(2\mu_x\dot\Phi_{,\xi\xi} + (1-\nu)\alpha_1^2\mu_{xy}\dot\Phi_{,\eta\eta}) + \alpha_1^2 K_8(2\nu\mu_y + (1-\nu)\mu_{xy})\dot\Psi_{,\xi\eta} + \nu K_9 \dot W_{,\xi}$$
$$+ \alpha_3 K_7(2\Phi\Phi_{,\xi} + 2\nu\alpha_1^2\Psi\Psi_{,\xi} + (1-\nu)\alpha_1^2(\Phi_{,\eta}\Psi + \Phi\Psi_{,\eta})) + K_{10}(2W_{,\xi}W_{,\xi\xi}$$
$$+ 2\nu\alpha_1^2 W_{,\eta}W_{,\xi\eta} + (1-\nu)\alpha_1^2(W_{,\xi\eta}W_{,\eta} + W_{,\xi}W_{,\eta\eta})) + \alpha_3 K_7(2(\mu_x(\Phi_{,\xi}\dot\Phi + \Phi\dot\Phi_{,\xi})$$
$$+ \nu\alpha_1^2\mu_y(\Psi_{,\xi}\dot\Psi + \Psi\dot\Psi_{,\xi})) + (1-\nu)\alpha_1^2\mu_{xy}(\dot\Phi_{,\eta}\Psi + \dot\Phi\Psi_{,\eta} + \Phi_{,\eta}\dot\Psi + \Phi\dot\Psi_{,\eta}))$$
$$+ K_{11}\ddot U + K_{12}\ddot\Phi + K_{13}\ddot W_{,\xi} + K_{1T} = 0.$$

$$K_2 U_{,\xi\eta} + K_1(2\alpha_1^2 V_{,\eta\eta} + (1-\nu)V_{,\xi\xi}) + K_4\Phi_{,\xi\eta} + \alpha_3 K_3(2\alpha_1^2\Psi_{,\eta\eta} + (1-\nu)\Psi_{,\xi\xi})$$
$$+ \alpha_1 K_5(W_{,\xi\xi\eta} + \alpha_1^2 W_{,\eta\eta\eta}) + \alpha_1 K_6 W_{,\eta} + \alpha_1 K_7(2\nu\mu_x + (1-\nu)\mu_{xy})\dot U_{,\xi\eta} + K_7(\alpha_1^2\mu_y\dot V_{,\eta\eta}$$
$$+ (1-\nu)\mu_{xy}\dot V_{,\xi\xi}) + \alpha_1 K_8(2\nu\mu_x + (1-\nu)\mu_{xy})\dot\Phi_{,\xi\eta} + \alpha_1 K_8(2\alpha_1^2\mu_y\dot\Psi_{,\eta\eta} + (1-\nu)\mu_{xy}\dot\Psi_{,\xi\xi})$$
$$+ \alpha_1 K_9\dot W_{,\eta} + \alpha_1\alpha_3 K_7(2\nu\Phi\Phi_{,\eta} + 2\alpha_1^2\Psi\Psi_{,\eta} + (1-\nu)(\Phi_{,\xi}\Psi + \Phi\Psi_{,\xi}))$$
$$+ \alpha_1 K_{10}(2\alpha_1^2 W_{,\eta}W_{,\eta\eta} + 2\nu W_{,\xi}W_{,\xi\eta} + (1-\nu)(W_{,\xi\xi}W_{,\eta} + W_{,\xi}W_{,\xi\eta}))$$
$$+ \alpha_1\alpha_3 K_7(2(\nu\mu_x(\Phi_{,\eta}\dot\Phi + \Phi\dot\Phi_{,\eta}) + \alpha_1^2\mu_y(\Psi_{,\eta}\dot\Psi + \Psi\dot\Psi_{,\eta})) + (1-\nu)\mu_{xy}(\dot\Phi_{,\xi}\Psi$$
$$+ \dot\Phi\Psi_{,\xi} + \Phi_{,\xi}\dot\Psi + \Phi\dot\Psi_{,\xi})) + K_{11}\ddot V + \alpha_1 K_{12}\ddot\Psi + \alpha_1 K_{13}\ddot W_{,\eta} + K_{2T} = 0.$$

$$K_3(2U_{,\xi\xi} + (1-\nu)\alpha_1^2 U_{,\eta\eta}) + K_4 V_{,\xi\eta} + K_{14}(2\Phi_{,\xi\xi} + (1-\nu)\alpha_1^2\Phi_{,\eta\eta}) + (K_{16} + K_{3T})\Phi + K_{15}\Psi_{,\xi\eta}$$
$$+ K_{18}(W_{,\xi\xi\xi} + \alpha_1^2 W_{,\xi\eta\eta}) - K_{16}W_{,\xi} + \nu K_{19}W_{,\xi} - K_8(2\mu_x\dot U_{,\xi\xi} + (1-\nu)\mu_{xy}\alpha_1^2\dot U_{,\eta\eta})$$
$$+ \alpha_1 K_8(2\nu\mu_x + (1-\nu)\mu_{xy})\dot V_{,\xi\eta} + K_{20}(2\mu_x\dot\Phi_{,\xi\xi} + (1-\nu)\alpha_1^2\mu_{xy}\dot\Phi_{,\eta\eta}) + \mu_{xz}K_{16}\dot\Phi$$
$$+ \alpha_1^2 K_{20}(\nu\mu_y + (1-\nu)\mu_{xy})\dot\Psi_{,\xi\eta} - \mu_{xz}K_{16}\dot W_{,\xi} - \alpha_3 K_7(2U_{,\xi}\Phi + 2\nu\alpha_1 V_{,\eta}\Phi + (1-\nu)\alpha_1^2 U_{,\eta}\Psi$$
$$+ (1-\nu)\alpha_1 V_{,\xi}\Psi) + K_{21}(\Psi\Psi_{,\xi} - \Psi_{,\eta}\Phi) + \nu K_{22}W\Phi + K_{23}(2W_{,\xi}W_{,\xi\xi} + 2\nu\alpha_1^2 W_{,\eta}W_{,\xi\eta}$$
$$+ (1-\nu)\alpha_1^2(W_{,\xi\eta}W_{,\eta} + W_{,\xi}W_{,\eta\eta})) - \alpha_3 K_7(2\mu_x\dot U_{,\xi}\Phi - (1-\nu)\alpha_1^2\mu_{xy}\dot U_{,\eta}\Psi + 2\nu\alpha_1\mu_y\dot V_{,\eta}\Phi$$
$$+ (1-\nu)\alpha_1\mu_{xy}\dot V_{,\xi}\Psi) + \alpha_3 K_8(2\mu_x\Phi_{,\xi}\dot\Phi + 2\nu\mu_y(\Psi_{,\xi}\dot\Psi + \Psi\dot\Psi_{,\xi}) - 2\nu\alpha_1^3\mu_y\dot\Psi_{,\eta}\Phi$$
$$- (1-\nu)\alpha_1^2\mu_{xy}(\dot\Phi\Psi_{,\eta} + \Phi_{,\eta}\dot\Psi + \Phi\dot\Psi_{,\eta} + \dot\Psi_{,\eta}\Psi)) - \nu\alpha_3 K_9\dot W\Phi - \alpha_3^2 K_7(\Phi^3 + \alpha_1^2\Psi^2\Phi$$
$$+ 2(\mu_x\dot\Phi\Phi^2 + \nu\mu_y\alpha_1^2\Psi\Phi\dot\Psi) + (1-\nu)\alpha_1^2\mu_{xy}(\dot\Phi\Psi^2 + \Phi\Psi\dot\Psi)) + K_{12}\ddot U + K_{24}\ddot\Phi + K_{25}\ddot W_{,\xi} + K_{4T} = 0,$$

$$\alpha_1 K_4 U_{,\xi\eta} + \alpha_1 K_3(2\alpha_1^2 V_{,\eta\eta} + (1-\nu)V_{,\xi\xi}) + K_{15}\Phi + \alpha_1^2 K_{14}((1-\nu)\Psi_{,\xi\xi} + 2\alpha_1^2\Psi_{,\eta\eta})$$
$$+ (K_{17} + \alpha_1 K_{3T})\Psi + \alpha_1^2 K_{18}(W_{,\xi\xi\eta} + \alpha_1^2 W_{,\eta\eta\eta}) - K_{17}W_{,\eta} + \alpha_1^2 K_{19}W_{,\eta} + \alpha_1^2 K_8(2\nu\mu_x + (1-\nu)\mu_{xy})\dot U_{,\xi\eta}$$
$$+ \alpha_1 K_8(2\mu_y\alpha_1^2\dot V_{,\eta\eta} + (1-\nu)\mu_{xy}\dot V_{,\xi\xi}) + \alpha_1^2 K_{20}(2\nu\mu_x + (1-\nu)\mu_{xy})\dot\Phi_{,\xi\eta} + \alpha_1^2 K_{20}(2\alpha_1^2\mu_y\dot\Psi_{,\eta\eta}$$
$$+ (1-\nu)\mu_{xy}\dot\Psi_{,\xi\xi}) + \mu_{yz}K_{17}\dot\Psi - \mu_{yz}K_{17}\dot W_{,\eta} - \alpha_1\alpha_3 K_7(2\nu\alpha_1 U_{,\xi}\Psi + (1-\nu)\alpha_1 U_{,\eta}\Phi + 2\alpha_1^2 V_{,\eta}\Psi$$
$$+ (1-\nu)V_{,\xi}\Phi) + K_{21}(\Phi\Phi_{,\eta} - \Phi_{,\xi}\Psi) + \alpha_1^2 K_{22}W\Psi + \alpha_1^2 K_{23}(2\alpha_1^2 W_{,\eta}W_{,\eta\eta} + 2\nu W_{,\xi}W_{,\xi\eta}$$
$$+ (1-\nu)(W_{,\xi\xi}W_{,\eta} + W_{,\xi}W_{,\xi\eta})) - \alpha_1\alpha_3 K_7(2\nu\alpha_1\mu_x\dot U_{,\xi}\Psi + (1-\nu)\alpha_1\mu_{xy}\dot U_{,\eta}\Phi + 2\alpha_1^2\mu_y\dot V_{,\eta}\Psi$$
$$+ (1-\nu)\mu_{xy}\dot V_{,\xi}\Phi) + \alpha_1^2\alpha_3 K_8(2\nu\mu_x(\Phi_{,\eta}\dot\Phi + \Phi\dot\Phi_{,\eta}) + 2\alpha_1^2\mu_y\Psi_{,\eta}\dot\Psi - 2\nu\mu_x\dot\Phi_{,\xi}\Psi$$
$$- (1-\nu)\mu_{xy}(\dot\Phi_{,\xi}\Psi + \dot\Phi\Psi_{,\xi} + \Phi_{,\xi}\dot\Psi + \dot\Phi_{,\eta}\Phi)) - \alpha_1^2\alpha_3 K_9\dot W\Psi - \alpha_1^2\alpha_3^2 K_7(\Phi^2\Psi + \alpha_1^2\Psi^3$$
$$+ 2(\nu\mu_x\Phi\Psi\dot\Phi + \mu_y\Psi^2\dot\Psi) + (1-\nu)\mu_{xy}(\Phi\dot\Phi\Psi + \Phi^2\dot\Psi)) + \alpha_1 K_{12}\ddot V + \alpha_1^2\ddot\Psi + \alpha_1^2 K_{25}\ddot W_{,\eta} + K_{5T} = 0,$$

$$- K_5(U_{,\xi\xi\xi} + \alpha_1^2 U_{,\xi\eta\eta}) - \nu K_6 U_{,\xi} - \alpha_1 K_5(V_{,\xi\xi\eta} + \alpha_1^2 V_{,\eta\eta\eta}) - \alpha_1 K_6 V_{,\eta} - K_{18}(\Phi_{,\xi\xi\xi} + \alpha_1^2\Phi_{,\xi\eta\eta})$$
$$+ (K_{16} + \nu K_{19})\Phi_{,\xi} - \alpha_1^2 K_{18}(\Psi_{,\xi\xi\eta} + \alpha_1^2\Psi_{,\eta\eta\eta}) + (K_{17} + \alpha_1^2 K_{19})\Psi_{,\eta} + K_{26}(W_{,\xi\xi\xi\xi} + 2\alpha_1^2 W_{,\xi\xi\eta\eta}$$
$$+ \alpha_1^4 W_{,\eta\eta\eta\eta}) - K_{16}W_{,\xi\xi} - K_{17}W_{,\eta\eta} + K_{27}(\alpha_1^2 W_{,\eta\eta} + \nu W_{,\xi\xi}) + K_{28}W + K_{6T}W_{,\xi} + K_{7T}W_{,\eta}$$
$$+ K_{8T}(W_{,\xi\xi} + \alpha_1^2 W_{,\eta\eta}) + K_{29}\dot U_{,\xi} - \alpha_1 K_9\dot V_{,\eta} + \mu_{xz}K_{16}\dot\Phi_{,\xi} + \mu_{yz}K_{17}\dot\Psi_{,\eta} - \mu_{xz}K_{16}\dot W_{,\xi\xi} - \mu_{yz}K_{17}\dot W_{,\eta\eta}$$

$$
\begin{aligned}
&+K_{29}\dot{W}-K_{10}(2(U_{,\xi\xi}W_{,\xi}+U_{,\xi}W_{,\xi\xi})+2\nu\alpha_1^2(U_{,\xi\eta}W_{,\eta}+U_{,\xi}W_{,\eta\eta})+(1-\nu)\alpha_1^2(U_{,\eta\eta}W_{,\xi}+2U_{,\eta}W_{,\xi\eta}\\
&+U_{,\xi\eta}W_{,\eta}))-\alpha_1K_{10}(2\nu(V_{,\xi\eta}W_{,\xi}+V_{,\eta}W_{,\xi\xi})+2\alpha_1^2(V_{,\eta\eta}W_{,\eta}+V_{,\eta}W_{,\eta\eta})+(1-\nu)(V_{,\xi\eta}W_{,\xi}\\
&+2V_{,\xi}W_{,\xi\eta}+V_{,\xi\xi}W_{,\eta}))+K_{23}(2(\Phi_{,\xi\xi}W_{,\xi}+\Phi_{,\xi}W_{,\xi\xi})+2\nu\alpha_1^2(\Phi_{,\xi\eta}W_{,\eta}+\Phi_{,\xi}W_{,\eta\eta})\\
&+(1-\nu)\alpha_1^2(\Phi_{,\eta\eta}W_{,\xi}+2\Phi_{,\eta}W_{,\xi\eta}+\Phi_{,\xi\eta}W_{,\eta}))+\alpha_1^2K_{23}(2\nu(\Psi_{,\xi\eta}W_{,\xi}+\Psi_{,\eta}W_{,\xi\xi})\\
&+2\alpha_1^2(\Psi_{,\eta\eta}W_{,\eta}+\Psi_{,\eta}W_{,\eta\eta})+(1-\nu)(\Psi_{,\xi\eta}W_{,\xi}+2\Psi_{,\xi}W_{,\xi\eta}+\Psi_{,\xi\xi}W_{,\eta}))\\
&-2\nu\alpha_1^2\alpha_3K_5(W_{,\xi\eta}^2+W_{,\xi\xi}W_{,\eta\eta})+(1-\nu)\alpha_1^2\alpha_3K_5(W_{,\xi\eta}^2-W_{,\xi\xi}W_{,\eta\eta})+K_{30}(\nu(W_{,\xi}^2+2W_{,\xi\xi}W)\\
&+\alpha_1^2(W_{,\eta}^2+2W_{,\eta\eta}W))+\frac{\nu}{2}K_{22}\Phi^2+\tfrac{1}{2}\alpha_1^2K_{22}\Psi^2+\tfrac{1}{2}K_{22}(\nu\mu_x\Phi\dot{\Phi}+\mu_y\alpha_1^2\Psi\dot{\Psi})\\
&-\alpha_3K_{10}(3W_{,\xi}^2W_{,\xi\xi}+4\alpha_1^2W_{,\eta}W_{,\xi\eta}W_{,\xi}+3\alpha_1^4W_{,\eta}^2W_{,\eta\eta}+\alpha_1^2W_{,\eta}^2W_{,\xi\xi}+\alpha_1^2W_{,\xi}^2W_{,\eta\eta})\\
&-K_{13}\ddot{U}_{,\xi}-\alpha_1K_{13}\ddot{V}_{,\eta}-K_{25}\ddot{\Phi}_{,\xi}-\alpha_1^2K_{25}\ddot{\Psi}_{,\eta}+K_{31}(\ddot{W}_{,\xi\xi}+\alpha_1^2\ddot{W}_{,\eta\eta})+K_{32}\ddot{W}+K_{9T}=0, \qquad (9)
\end{aligned}
$$

where the coefficients K_i and K_{iT} are functions of the material properties, geometrical parameters and temperature. If there is no temperature change, the coefficients K_{iT} disappear. The coefficients in the above equations can be found in the Appendix. For simplicity the Poisson's ratios for three layers are assumed to have the same value $\nu_1 = \nu_2 = \nu_3 = \nu$. The diaeresis over U, V, Φ, Ψ and W denotes the second order differentiation of that variable with non-dimensional time τ. The dimensionless parameters used in eqn (9) are given by

$$
\begin{aligned}
&a/b=\alpha_1,\quad a/t_1=\alpha_2,\quad 1/\alpha_2=\alpha_3,\quad x/a=\zeta,\quad y/b=\eta,\\
&\alpha_2\phi=\Phi,\quad \alpha_2\psi=\Psi,\quad w/t_1=W,\quad u/t_1=U,\quad v/t_1=V,\\
&R/t_1=\tilde{R},\quad \rho_2/\rho_1=\Gamma_2,\quad \rho_3/\rho_1=\Gamma_3,\quad t_3/t_1=m,\quad t_2/t_1=k,\\
&G_2/E_1=\bar{G}_2,\quad E_3/E_1=n_3,\quad E_2/E_1=n_2,\quad \tau=\sqrt{[E_1/(\rho_1t_1^2)]}\,t. \qquad (10)
\end{aligned}
$$

3. TEMPERATURE EFFECT

A typical effect of temperature on the properties of the sandwich core materials (Young's and shear moduli and the coefficient of the viscous damping) is presented in Fig. 2. Generally, the curves for different materials have a similar shape. Three typical distinct

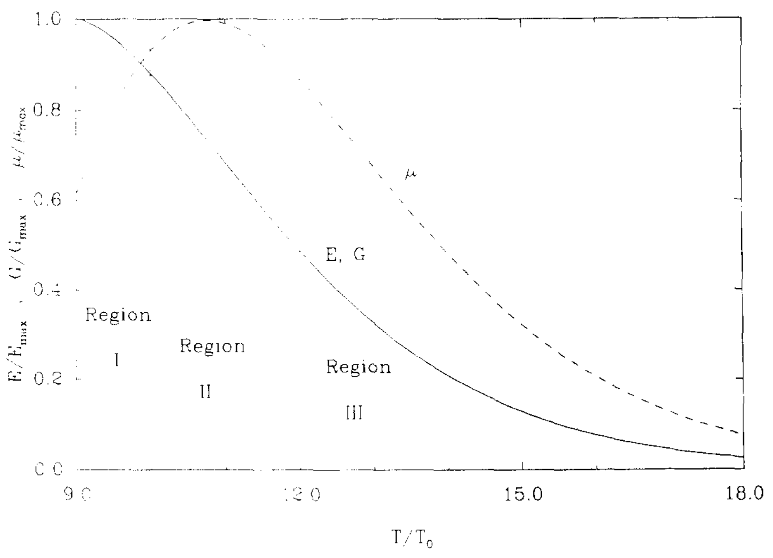

Fig. 2. Effect of temperature on the properties of sandwich core materials.

regions can be observed. Poisson's ratio depends weakly on the temperature change and is assumed to be a constant (Kamiya and Fukui, 1982). In the example discussed in this paper, the relationship of Young's and shear moduli and the damping coefficients versus temperature were approximated by the following simple formulae:

$$\frac{E}{E_{max}} = c_1\left(\frac{T-T_{Et}}{T_0}\right)^n e^{-k_t(T-T_{Et})/T_0}$$

$$\frac{G}{G_{max}} = c_1\left(\frac{T-T_{Gt}}{T_0}\right)^n e^{-k_t(T-T_{Gt})/T_0} \tag{11}$$

and

$$\frac{\mu}{\mu_{max}} = c_2\left(\frac{T-T_{\mu t}}{T_0}\right)^n e^{-k_t(T-T_{\mu t})/T_0}, \tag{12}$$

where T_0, T_{Et} and $T_{\mu t}$ are certain reference temperatures. The coefficients in eqns (11) and (12) are material dependent. The following values have been used in the numerical calculations for the example discussed:

$$n = 3, \quad c_1 = 0.385, \quad c_2 = 0.38, \quad k_t = 0.8, \quad \frac{T_{Et}}{T_0} = 5, \quad \frac{T_{\mu t}}{T_0} = 7. \tag{13}$$

The curves for E/E_{max}, G/G_{max} and μ/μ_{max} approximated using the above coefficients are presented in Fig. 2. They can simulate well the temperature-dependent behaviour of a typical damping material, for example, the material G.E. SMRD in the temperature range 100–200°F (Nashif *et al.*, 1985). The first region shown in Fig. 2 is characterized by an increase of the damping properties with the decrease of the value of the elastic modulus of the core. In the second region a further decrease of the elastic stiffness is observed while the damping factor takes its maximum value. The third region is the region where both the elastic moduli and the damping factor decrease.

When a panel is exposed to a changing temperature field, the temperature T at an arbitrary point of the panel can be approximated by the equation

$$T = T^1 + (2z/h)T^2, \tag{14}$$

with the resultant characteristic temperatures T^1 and T^2 assumed in the form

$$T^1 = \int_{-h/2}^{h/2} \lambda T \, dz, \quad T^2 = \int_{-h/2}^{h/2} \lambda T z \, dz. \tag{15}$$

In the above equations h is the thickness of the panel, z is the transverse dimension measured from the reference surface of the panel and λ is the corresponding coefficient of heat exchange. However, eqns (15) are difficult to use when T is still unknown. The temperatures T^1 and T^2 can be obtained by solving the following equations (Łukasiewicz, 1989):

$$K_T h \Delta T^1 - (12K_T/h)T^1 - C_v\rho\frac{\partial T^1}{\partial \tau} = -(6K_T/h)(T^+ + T^-)$$

$$K_T h \Delta T^2 - (60K_T/h)T^2 - C_v\rho\frac{\partial T^2}{\partial \tau} = -(30K_T/h)(T^+ - T^-), \tag{16}$$

where Δ is the Laplace operator, τ is non-dimensional time, K_T is the coefficient of heat conduction, C_v is the coefficient of the specific heat and ρ is mass density. T^+ and T^- are temperatures given at the upper and lower surfaces of the panel. The above equations are independent, linear, differential equations of second order and can be solved with respect to T^1 and T^2. With the approximation of eqn (16), the temperature defined in eqn (14) is linearly distributed across the thickness of the panel.

It was assumed in the example presented here that the external temperatures increase linearly with time, i.e.

$$T^+ = T_0^+ + T_t^+\tau, \quad T^- = T_0^- + T_t^-\tau. \tag{17}$$

T_0^+ and T_0^- are the reference temperatures for the upper and lower surfaces of the panel at time $\tau = 0$. T_t^+ and T_t^- present the rate of temperature change with time. If the temperature was uniformly distributed on the surfaces, then using eqns (17) and (16), the temperature in eqn (14) can be obtained in the following form:

$$\begin{aligned} T = {} & (0.5(T_0^+ + T_0^-)\mathrm{e}^{-\xi\tau} + 0.5((T_0^+ + T_0^-) - \xi^{-1}(T_t^+ + T_t^-))(1-\mathrm{e}^{-\xi\tau}) \\ & + 0.5(T_t^+ + T_t^-)\tau)\frac{2z}{h}(0.5(T_0^+ - T_0^-)\mathrm{e}^{-5\xi\tau} \\ & + 0.5((T_0^+ - T_0^-) - (5\xi)^{-1}(T_t^+ - T_t^-))(1-\mathrm{e}^{-5\xi\tau}) + 0.5(T_t^+ - T_t^-)\tau), \end{aligned} \tag{18}$$

where ξ is a material-dependent coefficient $(12K_T)/(\rho h^2 C_v)$. The heat transfer properties of the material play an important role in the temperature distribution. We observe that the temperatures in these layers do not change linearly with time. If T^+ and T^- are linear functions of time, the average temperature $\bar{T}$ in the panel can be approximated by (see the Appendix):

$$\bar{T} = A_1(\mathrm{e}^{-\xi\tau} + (1-\xi^{-1}A_2)(1-\mathrm{e}^{-\xi\tau}) + A_2\tau). \tag{19}$$

The coefficients A_1 and A_2 depend on the temperature field and material heat transfer properties.

The temperatures in the layers of a sandwich structure must be consistent. This means that the temperature T^+ of the ith layer is equal to the temperature T^- of the $(i+1)$th layer. The heat transfer property of the viscoelastic core is different from that of the facings. The average temperatures in three layers take the form:

$$\begin{aligned} \frac{\bar{T}_3}{T_0} &= (\mathrm{e}^{-175\tau} + 0.9772 \times (1-\mathrm{e}^{-175\tau}) + 4 \times \tau)\left(\frac{T}{T_0}\right)_0 \\ \frac{\bar{T}_2}{T_0} &= (2 \times \mathrm{e}^{-\tau} - 1 + 2 \times \tau)\left(\frac{T}{T_0}\right)_0 \\ \frac{\bar{T}_1}{T_0} &= (\mathrm{e}^{-175\tau} + 0.9972 \times (1-\mathrm{e}^{-175\tau}) + 0.5 \times \tau)\left(\frac{T}{T_0}\right)_0. \end{aligned} \tag{20}$$

The power of the exponent is very large as shown in above equations, due to the numerical value of the coefficient $(12K_T)/(\rho h^2 C_v)$ for typical materials. That means this term disappears very quickly with time, and later the equations are linear functions of time; but this term initially has a significant effect on temperature change. $(T/T_0)_0$ is the reference temperature ratio at time $\tau = 0$.

4. SOLUTION METHODOLOGY

The solution of eqn (9) was obtained for a simply supported cylindrical panel. In this case, the response functions which satisfy the boundary conditions can be chosen as follows:

$$
\begin{aligned}
U &= U(t)\cos(\pi\zeta)\sin(\pi\eta)\\
V &= V(t)\sin(\pi\zeta)\cos(\pi\eta)\\
\Phi &= \Phi(t)\cos(\pi\zeta)\sin(\pi\eta)\\
\Psi &= \Psi(t)\sin(\pi\zeta)\cos(\pi\eta)\\
W &= W(t)\sin(\pi\zeta)\sin(\pi\eta).
\end{aligned}
\tag{21}
$$

A set of five coupled, non-linear, ordinary differential equations was obtained for the time-dependent function, $U(t)$, $V(t)$, $\Phi(t)$, $\Psi(t)$, $W(t)$, by substituting eqns (21) into eqn (9) and multiplying the resulting equations by $\cos(\pi\zeta)\sin(\pi\eta)$, $\sin(\pi\zeta)\cos(\pi\eta)$, $\cos(\pi\zeta)\sin(\pi\eta)$, $\sin(\pi\zeta)\cos(\pi\eta)$ and $\sin(\pi\zeta)\sin(\pi\eta)$, respectively, and by integrating from zero to one with respect to ζ and η. This set of non-linear equations can be written as:

$$
\sum_{j=1}^{5} m_{ij}\ddot{a}_j(t) + \sum_{j=1}^{5} c_{ij}\dot{a}_j(t) + \sum_{j=1}^{5} k_{ij}a_j(t) + \sum_{j=1}^{5}\sum_{k=1}^{5} d_{ijk}a_j(t)a_k(t) + \sum_{j=1}^{5}\sum_{k=1}^{5} e_{ijk}a_j(t)\dot{a}_k(t)
$$
$$
+ \sum_{j=1}^{5}\sum_{k=1}^{5}\sum_{l=1}^{5} f_{ijkl}a_j(t)a_k(t)a_l(t) + \sum_{j=1}^{5}\sum_{k=1}^{5}\sum_{l=1}^{5} g_{ijkl}a_j(t)a_k(t)\dot{a}_l(t) - h_{iT} = 0
$$
$$
i, j, k, l = 1, \ldots, 5. \tag{22}
$$

Here, $a_j(t)$ are the generalized coordinates which represent $U(t)$, $V(t)$, $\Phi(t)$, $\Psi(t)$ and $W(t)$, respectively. m_{ij} are the inertia coefficients, c_{ij} the linear viscous damping coefficients and k_{ij} the linear stiffness coefficients. d_{ijk} and f_{ijkl} are quadratic and cubic non-linear stiffness coefficients, e_{ijk} and g_{ijkl} are quadratic and cubic non-linear damping coefficients, respectively and h_{iT} is the temperature equivalent loading. These temperature-dependent coefficients are not presented here. As can be seen from eqns (22), the non-linear damping is included in terms of quadratic and cubic forms. These kinds of equations were not found in the literature. The set of equations (22) was solved by the Runge–Kutta method which gave numerical results for U, V, Φ, Ψ and W and their rates $\dot{U}$, $\dot{V}$, $\dot{\Phi}$, $\dot{\Psi}$ and $\dot{W}$ at any time.

To measure the damping of the sandwich cylindrical panels, the dissipated energy [eqn (8)] is used. Introducing the obtained numerical results of U, V, Φ, Ψ and W and their rates $\dot{U}$, $\dot{V}$, $\dot{\Phi}$, $\dot{\Psi}$ and $\dot{W}$ from eqn (22) into relations (2)–(5) and then into (8), the change of dissipated energy with time can be studied (Figs 4–6).

5. RESULTS AND DISCUSSION

In the examples presented in this paper, the behaviour of plates and cylindrical panels is compared in a set of diagrams. The geometrical and material parameters used in this paper for both panels and plates have the same values at the initial time of vibration. The material properties change with time due to the change of temperature (see Fig. 2). It was assumed that the temperature changes with time according to eqns (20). The curvature $t_1/R = 0.01$ was used for the panel while $t_1/R = 0$ for plate.

$$
t_2/t_1 = 7,\quad t_3/t_1 = 2,\quad E_3/E_1 = 1,\quad E_2/E_1 = 0.002,\quad a/b = 2,\quad t_1/a = 0.005,
$$
$$
\rho_2/\rho_1 = 0.05,\quad \rho_3/\rho_1 = 1,\quad G_{xz}/E_1 = G_{yz}/E_1 = 0.04,\quad \nu_1 = \nu_2 = \nu_3 = 0.333,
$$
$$
\mu_{x} = \mu_{y} = 0.1\times10^{-4}\quad \rho_1 t_1^2/E_1 = 0.4\times10^{-13}.
$$

The basic values of the first layer made of steel are $E_1 = 2\times10^{11}\ \mathrm{N\,m^{-2}}$, $\rho_1 = 8\times10^{3}\ \mathrm{Ns^2\,m^{-4}}$ and $t_1 = 0.001$ m. A typical free vibration of a sandwich cylindrical panel based on the above parameters is presented in Fig. 3. It can be seen that the transverse deflection W changes with time irregularly. This phenomenon is due to the coupling of the in-plane displacements U and V with the transverse displacement W. It was found that when the material properties are temperature dependent, the change of temperature causes the change

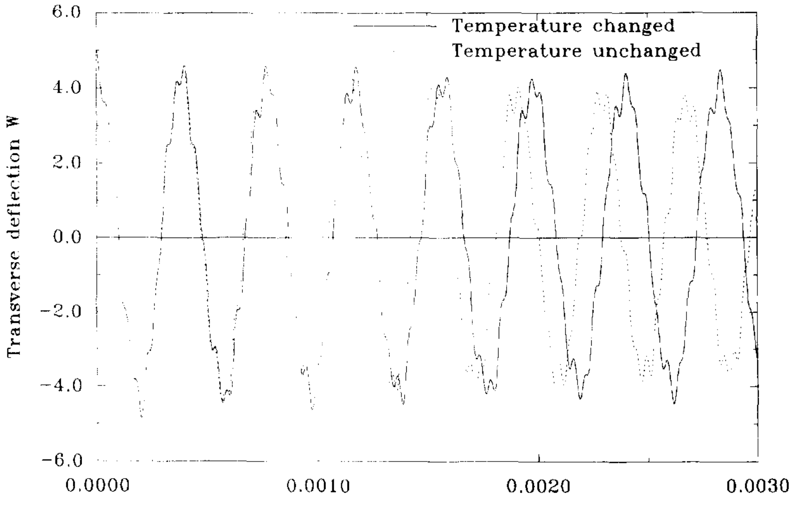

Fig. 3. Typical motion of a sandwich cylindrical panel.

of motion (Fig. 3). When the temperature increases with time, the material becomes softer. That results in a larger amplitude and smaller non-linear frequency of the motion for the panels.

Figures 4–6 show the effect of temperature on the damping properties of sandwich plates and panels in terms of dissipating energy. In all three figures, which correspond to the temperature changing in three different regions in Fig. 2, the ratios of dissipating energy to total energy are considerably different for structures in a constant temperature field or in a changing temperature field. In general, we can say that the ratio of the dissipating energy to the total energy for an unheated sandwich structure is always much smaller than that of a heated one. If the material properties change with temperature according to Fig. 2, the damping of the vibrating panel always increases with temperature even if the damping parameter μ_{ij} of the viscoelastic layer decreases in some regions. This phenomenon can be explained by the fact that the damping properties of the sandwich structure depend not only on the value of the damping parameter μ_{ij} of the second layer, but also on other material constants such as Young's modulus E and shear modulus G, the latter constants being even more important. A softer structure can result in a larger amplitude of vibration

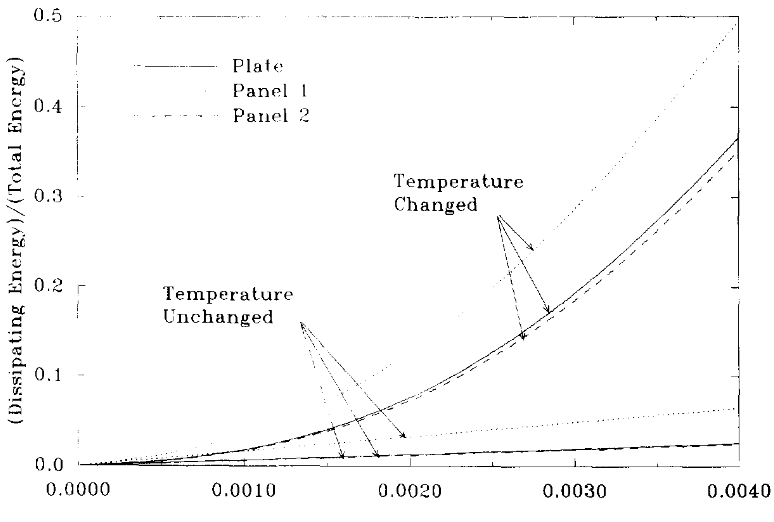

Fig. 4. Dissipating energy ratio versus time, material properties changing with temperature according to region I.

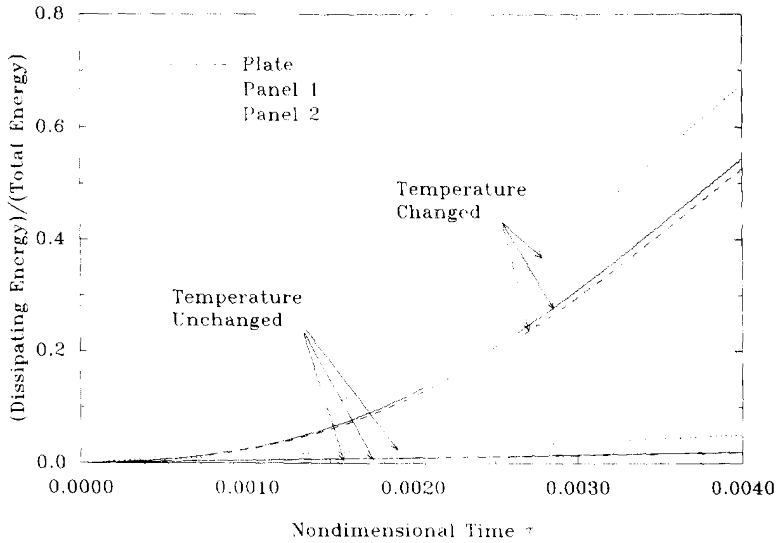

Fig. 5. Dissipating energy ratio versus time, material properties changing with temperature according to region II.

(Fig. 3), however it can also cause a larger dissipating energy ratio. The effect of the change of stiffness is more substantial than the change of the damping parameter μ. This can be concluded by comparing the results in Figs 4–6 with the result in Fig. 7. In Figs 4–6 the differences of the energy ratios for temperature affected and temperature unaffected structures are very large. In Fig. 7 the stiffnesses of the materials are assumed to be unaffected by the temperature, therefore only the change of the damping parameter μ_{ij} can affect the damping properties of the sandwich structure. The results shown in Fig. 7 indicate that the difference of energy ratio between temperature affected and unaffected structures is small in this case. When the temperature is assumed to be changed according to the third region of Fig. 2, the damping parameter μ_{ij} decreases with temperature. Therefore the total dissipating energy for the heated panels is always smaller than that of unheated ones in this temperature region; but a larger value of μ_{ij} can still cause more damping in the sandwich structure.

It is also noticed that panel 1 ($t_2/t_1 = 7$) is characterized by greater damping than panel 2 ($t_2/t_1 = 5$), which means that a thicker viscoelastic layer gives more damping to the

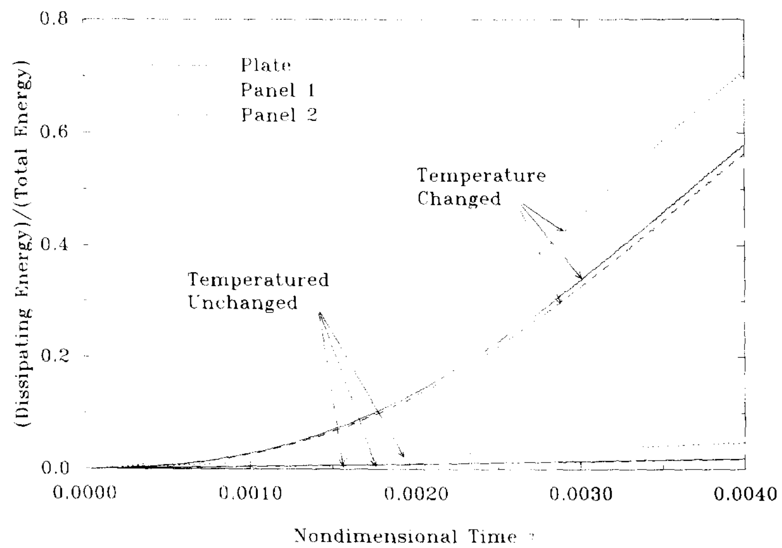

Fig. 6. Dissipating energy ratio versus time, material properties changing with temperature according to region III.

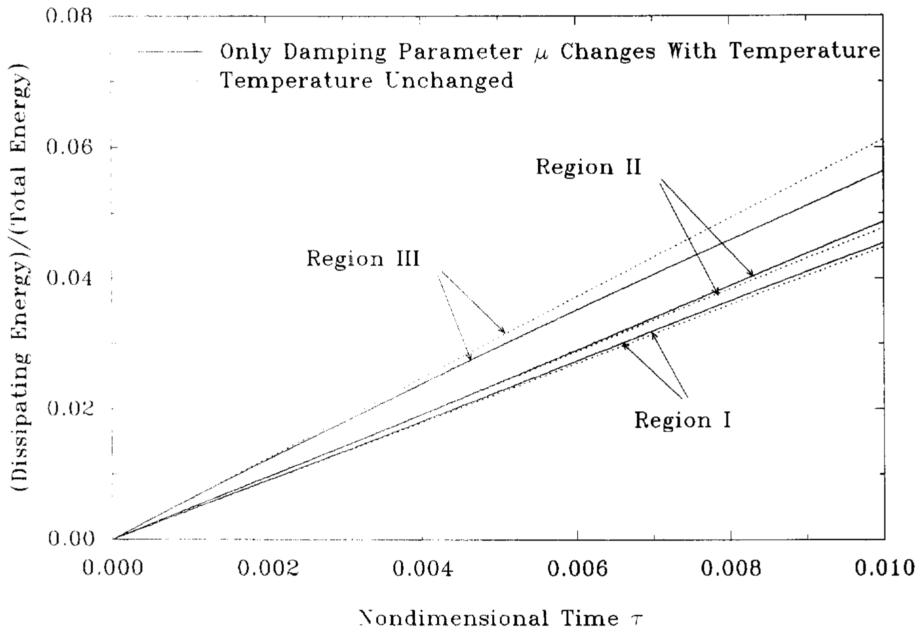

Fig. 7. Effect of damping parameter μ on dissipating energy ratio in three regions.

structure (Xia and Łukasiewicz. 1994. 1995). When the effect of the curvature is studied, it is also found that the plates show a stronger damping than panels. Larger curvature results in less damping.

6. CONCLUSION

As can be observed from the results obtained the effect of the temperature on the damping of sandwich structures is considerable. The major reason for the damping in the sandwich panels is the viscoelastic core. As the temperature affects the core, the damping of the structure increases when the temperature rises. However, at the same time the structural stiffness decreases. It was found that an increase of the damping is of lesser importance for the behaviour of the structure than the decrease of the stiffness. The damping increases with the increase of the temperature, but simultaneous softening of the structure can eventually cause larger amplitudes of vibrations.

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APPENDIX: COEFFICIENTS FOR EQN (9)

$$K_1 = -\frac{1}{4(1-\nu^2)}\alpha_3^2(2(1+n_3m+n_2k)+\frac{1}{\tilde{R}}(-(k+1)+(k+m)n_3m))$$

$$K_2 = \alpha_1(1-\nu)K_1$$

$$K_3 = \frac{1}{8(1-\nu^2)}k\alpha_3^3(2(1-n_3m)-\frac{1}{\tilde{R}}((k+1)+(k+m)n_3m+\frac{k}{3}n_2k))$$

$$K_4 = \alpha_1(1+\nu)K_3$$

$$K_5 = \frac{1}{4(1-\nu^2)}\alpha_3^3(2(1+n_3m^2)-\frac{1}{\tilde{R}}((1+\tfrac{4}{3})+(k+\tfrac{4}{3}m)n_3m^2))$$

$$K_6 = \frac{\nu}{(1-\nu^2)\tilde{R}}\alpha_3(1+n_3m+n_2k)$$

$$K_7 = -\frac{1}{2(1-\nu^2)}n_2k\alpha_3^3$$

$$K_8 = -\frac{k}{24(1-\nu^2)\tilde{R}}n_2k^2\alpha_3^3$$

$$K_9 = \frac{1}{(1-\nu^2)\tilde{R}}n_2k\alpha_3\mu$$

$$K_{10} = -\frac{1}{4(1-\nu^2)}\alpha_3^3(2(1+n_3m)-\frac{1}{\tilde{R}}((k+1)+(k+m)n_3m))$$

$$K_{11} = \tfrac{1}{2}\beta(2(1+m\Gamma_3)+\frac{1}{\tilde{R}}(-(k+1)+(k+m)m\Gamma_3-\tfrac{1}{3}k\Gamma_2))$$

$$K_{12} = -\tfrac{1}{4}\beta k\alpha_3(2(1-m\Gamma_3+k\Gamma_2)-\frac{1}{\tilde{R}}((k+1)+(k+m)m\Gamma_3))$$

$$K_{13} = -\tfrac{1}{2}\beta\alpha_3(2(1+m^2\Gamma_3)-\frac{1}{\tilde{R}}((k+\tfrac{4}{3})+(k+\tfrac{4}{3}m)m^2\Gamma_3))$$

$$K_{14} = -\frac{1}{16(1-\nu^2)}k^2\alpha_3^4(2(1+n_3m+n_2k)+\frac{1}{\tilde{R}}(-(k+1)+(k+m)n_3m))$$

$$K_{15} = \alpha_1^2(1+\nu)K_{14}$$

$$K_{16} = k\alpha_3^2\bar{G}_c$$

$$K_{17} = k\alpha_1^2\alpha_3^2\bar{G}_c$$

$$K_{18} = -\frac{1}{8(1-\nu^2)}k\alpha_3^4(2(1-n_3m^2)-\frac{1}{\tilde{R}}((k+\tfrac{4}{3})-(k+\tfrac{4}{3}m)n_3m^2))$$

$$K_{19} = \frac{1}{2(1-\nu^2)\tilde{R}}k\alpha_3^2(-1+n_3m)$$

$$K_{20} = -\frac{\tilde{R}}{8(1-\nu^2)}n_2k^2\alpha_3^4\ln\left(\frac{2\tilde{R}+k}{2\tilde{R}-k}\right)$$

$$K_{21} = \frac{(-1+3\nu)}{24(1-\nu^2)\tilde{R}} n_2 k^3 \alpha_1^2 \alpha_3^4$$

$$K_{22} = -\frac{1}{(1-\nu^2)\tilde{R}} n_2 k \alpha_3^2$$

$$K_{23} = \frac{1}{8(1-\nu^2)} k \alpha_3^4 (2(1-n_3 m) - \frac{1}{\tilde{R}}((k+1)+(k+m)n_3 m))$$

$$K_{24} = \tfrac{1}{8}\beta k^2 \alpha_3^2 (2(1+m\Gamma_3) - \frac{1}{\tilde{R}}((k+1)-(k+m)m\Gamma_3 + \tfrac{1}{3}k^2\Gamma_2))$$

$$K_{25} = \tfrac{1}{8}\beta k \alpha_3^2 (2(1-m^2\Gamma_3) - \frac{1}{\tilde{R}}((k+\tfrac{4}{3}) - (k+\tfrac{4}{3}m)m^2\Gamma_2))$$

$$K_{26} = \frac{1}{8(1-\nu^2)} \alpha_3^4 (2(1+n_3 m^3) + \frac{1}{\tilde{R}}(-(\tfrac{4}{3}k+2) + (\tfrac{4}{3}k+2m)n_3 m^3))$$

$$K_{27} = \frac{1}{(1-\nu^2)} \alpha_3^2 (1-n_3 m^3)$$

$$K_{28} = \frac{1}{(1-\nu^2)\tilde{R}}\left(\ln\left(\frac{2\tilde{R}-k}{2\tilde{R}-k-2}\right) - \ln\left(\frac{2\tilde{R}+k}{2\tilde{R}+k-2m}\right)n_3 m + \ln\left(\frac{2\tilde{R}+k}{2\tilde{R}-k}\right)n_2 k\right)$$

$$K_{29} = -\frac{\nu}{(1-\nu^2)\tilde{R}} n_2 k \alpha_3 \mu$$

$$K_{30} = \frac{1}{2(1-\nu^2)\tilde{R}} \alpha_3^2 (1+n_3 m)$$

$$K_{31} = -\tfrac{1}{8}\beta\alpha_3^2(2(1-m^3\Gamma_3) + \frac{1}{\tilde{R}}(-(\tfrac{4}{3}k+2) + (\tfrac{4}{3}k+2m)m^2\Gamma_3))$$

$$K_{32} = \tfrac{1}{2}\beta(2(1+m\Gamma_3+k\Gamma_2) + \frac{1}{\tilde{R}}(-(k+1)+(k+m)m\Gamma_3))$$

$$K_{1t} = \frac{1}{(1-\nu)\alpha_2}(\alpha_{t1}\bar{T}_{1,\xi} + n_3 m\alpha_{t3}\bar{T}_{3,\xi} + n_2 k\alpha_{t2}\bar{T}_{2,\xi})$$

$$K_{2t} = \frac{1}{(1-\nu)\alpha_2}(\alpha_{t1}\bar{T}_{1,\eta} - n_3 m\alpha_{t3}\bar{T}_{3,\eta} + n_2 k\alpha_{t2}\bar{T}_{2,\eta})$$

$$K_{3t} = -\frac{n_2 k\alpha_{t2}}{2(1-\nu)\alpha_2}\bar{T}_2$$

$$K_{4t} = \frac{k}{2(1-\nu)\alpha_2}(-\alpha_{t1}\bar{T}_{1,\xi} + n_3 m\alpha_{t3}\bar{T}_{1,\xi} - 2n_2 k\alpha_{t2}\tilde{T}_{2,\xi})$$

$$K_{5t} = \frac{k\alpha_1}{2(1-\nu)\alpha_2}(-\alpha_{t1}\bar{T}_{1,\eta} + n_3 m\alpha_{t3}\bar{T}_{1,\eta} - n_2 k\alpha_{t2}\tilde{T}_{2,\eta})$$

$$K_{6t} = \frac{1}{(1-\nu)\alpha_2^2}(\alpha_{t1}\bar{T}_{1,\xi} - n_3 m\alpha_{t3}\tilde{T}_{3,\xi})$$

$$K_{7t} = \frac{\alpha_1^2}{(1-\nu)\alpha_2^2}(\alpha_{t1}\bar{T}_{1,\eta} + n_3 m\alpha_{t3}\tilde{T}_{3,\eta})$$

$$K_{8t} = \frac{1}{(1-\nu)\alpha_2^2}(\alpha_{t1}\bar{T}_1 + n_3 m\alpha_{t3}\bar{T}_3)$$

$$K_{9t} = -\frac{k}{2(1-\nu)\alpha_2^2}[\alpha_{t1}(\bar{T}_{1,\xi\xi} + \bar{T}_{1,\eta\eta}) + n_3 m\alpha_{t3}(\bar{T}_{3,\xi\xi} + \bar{T}_{3,\eta\eta})] + \frac{\alpha_1^2}{1-\nu}[\alpha_{t1}(\tilde{T}_{1,\xi\xi} + \tilde{T}_{1,\eta\eta})$$
$$+ n_3 m^2\alpha_{t3}(\tilde{T}_{3,\xi\xi} + \tilde{T}_{3,\eta\eta})] + \frac{1}{(1-\nu)\tilde{R}}(\alpha_{t1}T_1^m + n_3 m\alpha_{t3}T_3^m + n_2 k\alpha_{t2}T_2^m),$$

where

$$T_i^{m} = \frac{1}{t_i}\int T_i(\tau)\,\mathrm{d}z,$$

$$\bar{T}_i = \frac{1}{t_i}\int \frac{(R-z)}{R}\,T_i(\tau)\,\mathrm{d}z,$$

$$\tilde{T}_i = \frac{1}{t_i^2}\int \frac{(R-z)z}{R}\,T_i(\tau)\,\mathrm{d}z.$$

τ is time. $T_i(\tau)$ is the time-dependent temperature in the ith layer and t_i is the thickness of the ith layer.